Written Exam for the M.Sc. in Economics Winter 2012-2013

ADVANCED MACROECONOMETRICS

Final Exam
January 23, 10:00 – January 25, 10:00

PLEASE NOTE that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The paper must be uploaded as one PDF document (including the standard cover and the appendices). The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Absalon.

FOCUS ON EXAM CHEATING: In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

Practical Information

Note the following formal requirements:

- This is an *individual* examination. You are not allowed to cooperate with other students or other people, see the *focus on exam cheating* above.
- The assignment consists of Sections 1-5 with 21 questions to be answered. *Please answer all questions.*
- The exam paper should not exceed 20 pages. A maximum of 20 pages of supporting material (graphs, estimation output, etc.) can accompany the paper as appendices. You may refer to the computer output in the appendices when answering the questions. Also, you may add clarifying comments in the output as part of your answer.
- All pages must be numbered consecutively and marked with your *exam number*. You should *not* write your name on the exam paper.
- *Your paper must be uploaded on the course page in Absalon at the given time.* The exam paper (including supporting material) must be in *PDF-format* and collected in *one file only*; the uploaded file must be named `1234.pdf`, where `1234` is your exam number.

The purpose of the examination is to assess your understanding of the cointegrated VAR (CVAR) model, your ability to use statistical procedures to make inference on the equilibrium structures and the dynamic adjustment properties, as well as your ability to interpret the results. Most questions in the examination are applied, concerning the empirical example outlined below. When you answer these empirical questions, please explain and motivate your answer as detailed as possible, preferably with reference to the underlying theory.

Regarding the data for the exam paper, please note the following:

- All assignments are based on *different* data sets. You should use the data set located in the Excel file `Data1234.xls`, where `1234` is your exam number.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

1 Background and Statistical Model

This project examination deals with interest rate linkages between countries in a monetary union. The monetary union was established on January 1^{st}, 1990, as a union between three countries, but it was enlarged in January 1997 such that it now consists of six countries. For the empirical analysis, consider time series for the long-term interest rates in the six participating countries,

$$x_t = \left(R_t^A : R_t^B : R_t^C : R_t^D : R_t^E : R_t^F\right)', \tag{1.1}$$

measured as returns from year to year in percentages and covering a period from 1990 to 2012. Country A is by far the largest and is the dominating country in the monetary union. The countries B and C also participated in the union from the start and are considered as *core members*. The three remaining countries, D, E and F, where closely associated to the monetary union from the start and tried to mimic the economic policy in the union. They joined the union in January 1^{st}, 1997.

You are informed that some large financial institutions in the countries collapsed during the summer of 2007. At that point, some commentators expressed fears, that this so-called *financial crisis* could permanently increase the perceived risk of assets denominated in the currency of the monetary union, and hence permanently change interest rate levels.

From the point of view of the monetary union, the most optimistic interpretation of the bond markets in different countries suggests that there is one underlying *factor*, that drives the long interest rates in the different countries. The central bank of the monetary union suggests the following *scenario* for the dynamics of the bond markets, i.e.

$$\begin{pmatrix} R_t^A \\ R_t^B \\ R_t^C \\ R_t^D \\ R_t^E \\ R_t^F \end{pmatrix} = \begin{pmatrix} 1 \\ \xi_B \\ \xi_C \\ \xi_D \\ \xi_E \\ \xi_F \end{pmatrix} \left(\sum_{i=1}^{t} u_{1i}\right) + S_{1t}, \tag{1.2}$$

where $\sum_{i=1}^{t} u_{1i}$ is an I(1) stochastic trend that affects all interest rates, and S_{1t} is a stationary process describing transitory movements in interest rates.

[1] Explain why the scenario in (1.2) could motivate the use of a cointegrated vector autoregression for the empirical analysis.
Explain what the structure would imply in terms of the cointegration rank and the structure of the cointegration space, and hence how the empirical relevance of this scenario could be tested.
What do you think the optimistic central bank would regard as reasonable parameters for ξ_i, $i = B, C, ..., F$?

Historically, market participants seem to have the perception that interest rates in the new member countries are less closely linked to the core of the union, and that their interest rates are driven also by a second stochastic trend, i.e.

$$\begin{pmatrix} R_t^A \\ R_t^B \\ R_t^C \\ R_t^D \\ R_t^E \\ R_t^F \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \xi_B & 0 \\ \xi_C & 0 \\ \xi_D & 1 \\ \xi_E & \eta_E \\ \xi_F & \eta_F \end{pmatrix} \begin{pmatrix} \sum_{i=1}^t u_{1i} \\ \sum_{i=1}^t u_{2i} \end{pmatrix} + S_{2t}, \tag{1.3}$$

where $\sum_{i=1}^t u_{2i}$ is the second trend pertaining to the new member countries and S_{2t} is a stationary process. An even more elaborate suggestion is that there is a third trend, driving the interest rates of the core member B and C.

[2] Modify the scenario in (1.3) to allow for a third trend driving the interest rates of country B and C, and state the implication in terms of the cointegration rank.
How would the scenarios have to be restricted if the spread between the core member countries is stationary, $(R_t^B - R_t^C) \sim I(0)$, and the spreads between the periphery countries are stationary, $(R_t^D - R_t^E) \sim I(0)$ and $(R_t^D - R_t^F) \sim I(0)$?

Now consider the $p-$dimensional process x_t and the vector autoregression:

$$x_t = \sum_{i=1}^{k} \Pi_i x_{t-i} + \phi D_t + \epsilon_t, \tag{1.4}$$

for $t = 1, 2, ..., T$, $\epsilon_t \sim N(0, \Omega)$, and initial values, $x_{-k+1}, ..., x_{-1}, x_0$, fixed. The vector D_t contains potential deterministic variables, such as a constant, a trend, and dummy variables relevant for the empirical analysis.

[3] Write the VAR(k) model in (1.4) as a companion form VAR(1) and state the conditions under which the VAR(k) model is stable, i.e. that time series generated from the equation are stationary.

[4] Set up and estimate a relevant empirical model for the data in (1.1) model. Carefully explain the steps you take and motivate the choices you make.
State all assumptions for the model, and test that the model is well specified. In practice it may not be possible to find a model that is acceptable in all directions, just do as well as you can.

[5] Check the stability conditions from question [3] for your preferred model. Comment on the results.

2 The Cointegration Rank

[6] State the relevant eigenvalue problem for maximum likelihood estimation of your preferred model above, i.e. for your preferred lag length and your chosen deterministic terms.

[7] Explain how the likelihood ratio test statistics for the reduced rank of $\Pi = \sum_{i=1}^{k} \Pi_i - I_p$ are calculated. Explain how the asymptotic distribution of the rank test statistic, involving Brownian motions, can be simulated using random walks. You should do this for your preferred model.

[8] Determine the cointegration rank, $r = \text{Rank}(\Pi)$, in your preferred model for the interest rates. How does that relate to the different scenarios presented above?

3 Hypotheses Testing

[9] Impose the reduced rank, $\Pi = \alpha\beta'$, and test for *long-run exclusion* for all variables in the model–including potential deterministic terms.

Explain why this restriction is not merely a normalization of β and explain how to calculate the degrees of freedom for the test.

[10] Next, test for the *stationarity* of some interest rate spreads, $R_t^i - R_t^j$, for $i, j \in \{A, B, C, D, E, F\}$, that you think are relevant with reference to the described scenarios. If relevant, also try to allow deterministic terms, $R_t^i - R_t^j + \rho D_t$.

Explain again how to calculate the degrees of freedom, and comment on the implications for the theoretical candidates.

For each accepted test, discuss the pattern of error correction and the economic interpretation.

[11] Test the hypothesis of weak exogeneity of each variable with respect to the cointegration parameters in β, and explain how the restriction of weak exogeneity affects the structure of the common stochastic trends.

[12] Test the hypothesis that each of the chocks in ϵ_t has only transitory effects on the variables in x_t.

[13] Explain how to test the hypothesis that one of the stochastic trends is composed by the average of shocks to the new member countries, D, E, and F:

$$ST_t = \sum_{i=1}^{t} (\epsilon_{D,t} + \epsilon_{E,t} + \epsilon_{F,t}),$$

where $ST_t = \sum_{i=1}^{t} u_{1t}$ is an I(1) stochastic trend. Perform the test.

4 Identification

Now we want to consider a restricted cointegration space,

$$\beta = (\beta_1 : \beta_2 : ... : \beta_r) = (H_1\varphi_1 : H_2\varphi_2 : ... : H_r\varphi_r),$$

where H_j is a known matrix and φ_j is a vector with parameters to be estimated, $j = 1, 2, ..., r$.

[14] Begin with a just identifying structure for β, inspired by the results you have obtained so far. State the relevant design matrices, H_j ($j = 1, 2, ..., r$), and check by the rank conditions that the system is generically identified.
Estimate the just identified model and comment on the results.

[15] Find the Granger representation for the model, and comment on the results. How do the results relate to the scenarios above? You may have to renormalize the common trends or their loadings to properly compare.

[16] Simplify the structure by imposing and testing over-identifying restrictions. Explain and motivate the route you take.
State your finally preferred model and the corresponding Granger representation and discuss the results.

[17] Perform a recursive estimation of the preferred identified structure and discuss the results. If you had found strong non-constancy, what would be your suggested remedy?

[18] Discuss any remaining aspects of the analysis that you find relevant, e.g. short-run of contemporaneous effects. Also highlight any remaining weakness of the model, or choices where you would have liked to evaluate the robustness.

5 Extensions

[19] (Causal Ordering) Consider three variables, $W_t = (x_t : y_t : z_t)'$, and the VAR model

$$W_t = \Pi_1 W_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \Omega). \tag{5.1}$$

Explain how you could perform an orthogonalized impulse response analysis, i.e. pictures of the dynamic impact on W_t of shocks to the variables, by using a Choleski decomposition. Explain why the ordering of the variables is important in this case.
Now you are informed that the estimated (conditional and unconditional) correlations are given by (with standard errors in parentheses)

$$\begin{aligned}
\text{corr}(x_t, y_t) &= \underset{(0.15)}{0.49} & \text{corr}(x_t, y_t \mid z_t) &= \underset{(0.17)}{0.53} \\
\text{corr}(y_t, z_t) &= \underset{(0.20)}{0.65} & \text{corr}(y_t, z_t \mid x_t) &= \underset{(0.23)}{0.70} \\
\text{corr}(x_t, z_t) &= \underset{(0.11)}{-0.03} & \text{corr}(x_t, z_t \mid y_t) &= \underset{(0.13)}{0.27}
\end{aligned}$$

Does that help you in determining a causal order? Motivate your answer as well as you can.

[20] (ASYMPTOTICS) Consider a univariate autoregression, AR(1), as given by

$$\Delta y_t = \delta + \pi y_{t-1} + \epsilon_t, \quad t = 1, 2, ..., T,$$

with $\epsilon_t \sim N(0, \sigma^2)$ and the initial value, y_0, given. In this model it holds that the LR-statistic for the null hypothesis of a unit root, $H_0 : \pi = 0$, has the property that, if the null hypothesis is true and as $T \to \infty$,

$$LR(\pi = 0) \xrightarrow{D} \begin{cases} \chi^2(1) & \text{if} \quad \delta \neq 0 \\ DF^2 & \text{if} \quad \delta = 0 \end{cases}$$

where DF^2 denotes the Dickey-Fuller type distribution, which is a function of Brownian motions.

Explain (*i*) why this property may be considered problematic, (*ii*) how it is related to the idea of similarity, and (*iii*) how it influences the suggested way of including deterministic variables in the CVAR model.

[21] (THRESHOLD VECTOR ERROR CORRECTION) A quite novel class of models (*not discussed during the lectures!*) suggests that the speed of adjustment depends on the magnitude of the deviation from equilibrium. Let R_t denote a regime switching variable that is equal to one if the deviation from equilibrium is numerically large and zero otherwise, i.e.

$$R_t = \begin{cases} 1 & \text{if} \quad |\beta' x_{t-1}| > \delta \\ 0 & \text{if} \quad |\beta' x_{t-1}| \leq \delta \end{cases} \tag{5.2}$$

for some value of the threshold parameter, δ. Now suppose we know the cointegration vector, $\beta = \bar{\beta}$, and the threshold parameter, $\delta = \bar{\delta}$. Then the vector error-correction model reads

$$\Delta x_t = R_t \cdot \alpha_1 \bar{\beta}' x_{t-1} + (1 - R_t) \cdot \alpha_2 \bar{\beta}' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \Omega),$$

where α_1 and α_2 are the adjustment parameters in the two regimes. *Based on your knowledge from the cointegrated VAR*, suggest a way to estimate the remaining parameters of the model $\theta = \{\alpha_1, \alpha_2, \Gamma_1, \Omega\}$.

If you try to estimate also δ, you will find that the likelihood function is not differentiable, due to the discrete function in (5.2). Explain why this could be a problem for the likelihood analysis.